May 13, 2014

Utility Maximization for Chunk-based OFDMA Systems with Multiple BER Requirements

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Prior Works

- Most of prior works is based on subcarrier-based allocation that individual subcarriers are assigned to a user. Yet, single-subcarrier based allocation can incur significant signaling overhead and entail complicated implementation.

- Recent work [Wang'11][Zhu'12]
  - Optimal chunk based strategies for utility maximization of average user rates under different power control policies.
  - Recent work by Zhu proposed a heuristic algorithm to minimize total transmit power for real-time data streams with multiple BER requirements.

Objective

- We investigate the optimal chunk-based allocation that maximizes a utility function of average user rates for OFDMA downlink, where data streams contain packets with diverse bit-error-rate (BER) requirements.

- Relying on adaptive transmission and stochastic tools, the proposed schemes with different α-fair utility functions can nicely balance total network throughput and fairness among users.

System Model

- Consider a downlink OFDMA system consisting of an access point (AP) and K wireless users $k = 1, \cdots, K$

- Total bandwidth B is divided into orthogonal narrowband, each with sub-bandwidth $\triangle f = B/(MJ)$

- Each chunk consists of J adjacent subcarriers.

- The data streams from AP to users contain packets with different quality of service requirements, e.g. video streams consist of base layers and enhancement layers, where reliable transmission of the base layer has a higher quality (smaller BER) requirement than enhancement layer.

- Relying on the channel conditions, L_b-ary quadrature amplitude modulation (QAM) is employed for adaptive modulation and coding (AMC) at the AP.

Problem Formulation

- α-fairness can be attained by the maximizer of a class of concave utility functions:

$$U_\alpha(x) = \begin{cases} x^{1-\alpha}/(1-\alpha), & \alpha \geq 0 \ \& \ \alpha \neq 1 \\ \ln(x), & \alpha = 1. \end{cases}$$

- We consider the following aggregate utility maximization problem:

$$\begin{aligned} \max_{\bar{\boldsymbol{R}} \geq \mathbf{0},\, \boldsymbol{\tau} \in \mathcal{T}} \quad & \sum_{k=1}^{K} U_\alpha(\bar{R}_k) \\ \text{s. t.} \quad & \bar{R}_k \leq \bar{r}_k(\boldsymbol{\tau}), \ \forall k, \\ & \bar{P}(\boldsymbol{\tau}) \leq \check{P}, \quad \bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\tau}) = \eta \bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\tau}), \ \forall k. \end{aligned}$$

where $\bar{\boldsymbol{R}} := \{\bar{R}_k, \forall k\}$ collect the achievable average user rates satisfying $\bar{R}_k \leq \bar{r}_k(\boldsymbol{\tau})$ for a certain feasible schedule $\boldsymbol{\tau}$.

$$\tau_{k,m,l}^{(q)}(\boldsymbol{\gamma}) \in \{0,1\}, \quad \sum_{k=1}^{K} \sum_{l=0}^{L} \sum_{q=0}^{1} \tau_{k,m,l}^{(q)}(\boldsymbol{\gamma}) = 1, \ \forall m.$$

Lagrange Dual Approach

- This problem is a mixed linear program (non-convex), yet it can be solved through dual-based Lagrange approach.

$$L(\boldsymbol{X}, \boldsymbol{\Lambda}) = \sum_{k=1}^{K} U(\bar{R}_k) - \sum_{k=1}^{K} \nu_k [\bar{R}_k - \bar{r}_k(\boldsymbol{\tau})]$$
$$- \lambda[\bar{P}(\boldsymbol{\tau}) - \check{P}] - \sum_{k=1}^{K} \mu_k [\bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\tau}) - \eta \bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\tau})]$$

- Then, Lagrange dual function is given by: $D(\boldsymbol{\Lambda}) = \min_{\boldsymbol{X}} \; L(\boldsymbol{X}, \boldsymbol{\Lambda}),$

- And the dual problem is: $\max_{\boldsymbol{\Lambda} \geq 0} \; D(\boldsymbol{\Lambda}).$

- To solve dual problem, we need to specify the dual function by defining:

$$\varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma}) := \begin{cases} \nu_k J\rho_l - \lambda J\pi_{k,m,l}^{(0)}(\boldsymbol{\gamma}) - \mu_k J\rho_l, & q=0 \\ \nu_k J\rho_l - \lambda J\pi_{k,m,l}^{(1)}(\boldsymbol{\gamma}) + \mu_k \eta J\rho_l, & q=1 \end{cases}$$

- Then, we can rewrite the Lagrange function as follows:

$$L(\boldsymbol{X},\mathbf{\Lambda}) = \lambda\check{P} + \sum_{k=1}^{K}[U(\bar{R}_k) - \nu_k\bar{R}_k] + \mathbb{E}_{\boldsymbol{\gamma}}\Big[\sum_{m=1}^{M}\Big\{\sum_{k,l,q}\tau_{k,m,l}^{(q)}(\boldsymbol{\gamma})\varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma})\Big\}\Big].$$

- We can adopt a “winner-takes-all” strategy that chunk m is assigned to a triplet

$$\{k_m^*, l_m^*, q_m^*\}(\mathbf{\Lambda};\boldsymbol{\gamma}) = \arg\max_{(k,l,q)} \varphi_{k,m,l}^{(q)}(\mathbf{\Lambda};\boldsymbol{\gamma}), \quad \forall m.$$

- The dual problem can be solved through the following (sub-)gradient descent iterations:

$$\nu_k[n+1] = \left[\nu_k[n] + \beta\left(\bar{R}_k^*(\mathbf{\Lambda}[n]) - \bar{r}_k^*(\boldsymbol{\tau}^*(\mathbf{\Lambda}[n]))\right)\right]^+,$$
$$\lambda[n+1] = \left[\lambda[n] + \beta\left(\bar{P}(\boldsymbol{\tau}^*(\mathbf{\Lambda}[n])) - \check{P}\right)\right]^+,$$
$$\mu_k[n+1] = \mu_k[n] + \beta\left(\bar{r}_k^{\mathrm{HQ}}(\boldsymbol{\tau}^*(\mathbf{\Lambda}[n])) - \eta\bar{r}_k^{\mathrm{LQ}}(\boldsymbol{\tau}^*(\mathbf{\Lambda}[n]))\right)$$

Stochastic Iterations

- To implement sub-gradient iteration, we need the knowledge of fading channel cdf. However, the practical mobile applications need to be operated without the knowledge of channel cdf by "learning" channel statistics on-the-fly.

- After transmissions, let $P_m(\boldsymbol{\Lambda};\boldsymbol{\gamma}[n]) = J\pi^{(q_m^*)}_{k_m^*,m,l_m^*}(\boldsymbol{\Lambda};\boldsymbol{\gamma}[n])$ collect the values of instantaneous powers per chunk, and instantaneous HQ-LQ packet rate balance at per chunk per user:

$$Q_{k,m}(\boldsymbol{\Lambda};\boldsymbol{\gamma}[n]) = \begin{cases} J\rho_{l_m^*} & \text{if } k = k_m^* \ \&\ q_m^* = 0; \\ -\eta J\rho_{l_m^*} & \text{if } k = k_m^* \ \&\ q_m^* = 1; \\ 0 & \text{if } k \neq k_m^*; \end{cases}$$

- In addition $\{P_m(\boldsymbol{\Lambda};\boldsymbol{\gamma}[n]), Q_{k,m}(\boldsymbol{\Lambda};\boldsymbol{\gamma}[n])\}$, the instantaneous rate per chunk per user:

$$R_{k,m}(\boldsymbol{\Lambda};\boldsymbol{\gamma}[n]) = \begin{cases} J\rho_{l_m^*} & \text{if } k = k_m^*, \\ 0 & \text{if } k \neq k_m^*; \end{cases}$$

- To this end, the AP can perform the stochastic iteration:

$$\hat{\nu}_k[n+1] = [\hat{\nu}_k[n] + \beta(\bar{R}_k^*(\hat{\mathbf{\Lambda}}[n]) - \sum_{m=1}^{M} R_{k,m}(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n]))]^+$$

$$\hat{\lambda}[n+1] = [\hat{\lambda}[n] + \beta(\sum_{m=1}^{M} P_m(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n]) - \check{P})]^+$$

$$\hat{\mu}_k[n+1] = \hat{\mu}_k[n] + \beta(\sum_{m=1}^{M} Q_{k,m}(\hat{\mathbf{\Lambda}}[n];\boldsymbol{\gamma}[n]))$$

- The companion chunk allocation can asymptotically converge to the optimal solution without a-priori knowledge of the fading channel cdf.

- This on-line scheduling algorithm can be implemented as fast convergent, asymptotically optimal, and has a low complexity of $\mathcal{O}(2KML)$ per slot.

α- Fair Utility Maximization

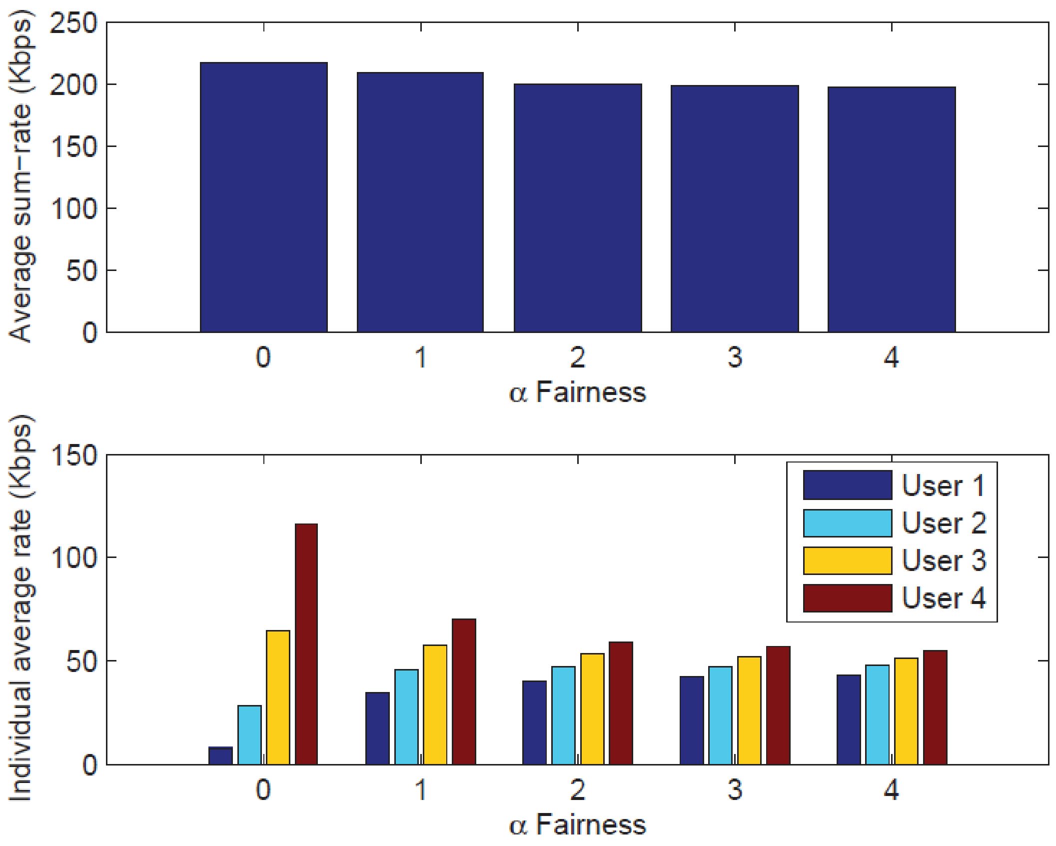

Fig. 1. Average sum-rates of all users (top) and the average rates for individual users (bottom).

Evolution of Lagrange multipliers

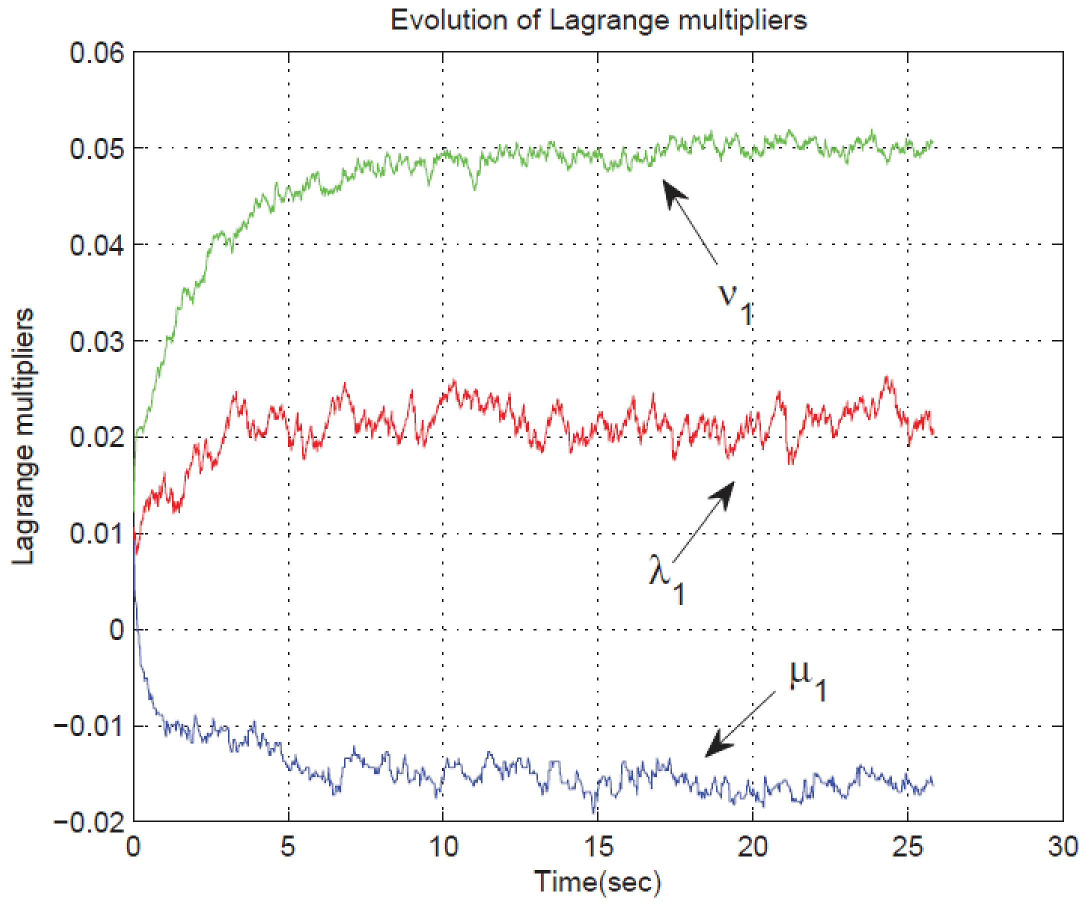

Fig. 2. Evolution of Lagrange multipliers for utility maximization.

Conclusions

- We formulated and solved optimal chunk-based allocation that maximizes a utility function of average user rates for OFDMA downlink, where data streams contain packets with diverse bit-error-rate (BER) requirements

- The stochastic approach was adopted to develop a novel on-line algorithm that approach to the optimal benchmark with low linear complexity.

- The proposed optimal chunk allocation schemes with different α-fair utility functions can nicely trade off total network throughput for fairness among users.

Thank You